

LEFT-ORDERABLE FUNDAMENTAL GROUP AND DEHN SURGERY ON THE KNOT 5_2

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ABSTRACT. We show that the resulting manifold by r -surgery on the knot 5_2 , which is the two-bridge knot corresponding to the rational number $3/7$, has left-orderable fundamental group if the slope r satisfies $0 \leq r \leq 4$.

1. INTRODUCTION

A group G is said to be *left-orderable* if it admits a strict total ordering, which is left invariant. More precisely, this means that if $g < h$ then $fg < fh$ for any $f, g, h \in G$. The fundamental groups of many 3-manifolds are known to be left-orderable. On the other hand, the fundamental groups of lens spaces are not left-orderable, because any left-orderable group is torsion-free. The notion of an L -space is introduced by Ozsváth and Szabó [12] in terms of Heegaard-Floer homology. Lens spaces, Seifert fibered manifolds with finite fundamental groups are typical examples of L -spaces. Although it is an open problem to give a topological characterization of an L -space, there is a conjectured connection between L -spaces and left-orderability. More precisely, Boyer, Gordon and Watson [3] conjecture that an irreducible rational homology sphere is an L -space if and only if its fundamental group is not left-orderable. They give affirmative answers for several classes of 3-manifolds.

It is well known that all knot groups are left-orderable (see [4]), but the resulting closed 3-manifold by Dehn surgery on a knot does not necessarily have a left-orderable fundamental group. For examples, there are many knots which admit Dehn surgery yielding lens spaces. By [12], the figure-eight knot has no Dehn surgery yielding L -spaces. Hence we can expect that any non-trivial surgery on the figure-eight knot yields a manifold whose fundamental group is left-orderable, if we support the conjecture above. In fact, Boyer, Gordon and Watson [3] show that if $-4 < r < 4$, then r -surgery on the figure-eight knot yields a manifold whose fundamental group is left-orderable. In addition, Clay, Lidman and Watson [6] verified it for $r = \pm 4$ through a different argument.

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In this paper, we follow the argument of [3] for the most part to handle the knot 5_2 in the knot table (see [14]). This knot is the two-bridge knot corresponding to the rational number $3/7$, which is a twist knot. We believe that this is an appropriate target next to the figure-eight knot. Since 5_2 is non-fibered, it does not admit Dehn surgery yielding an L -space [11]. Hence we can expect again that any non-trivial Dehn surgery on 5_2 yields a 3-manifold whose fundamental group is left-orderable.

Theorem 1.1. *Let K be the knot 5_2 . If $0 \leq r \leq 4$, then r -surgery on K yields a manifold whose fundamental group is left-orderable.*

In fact, 0-surgery on any knot yields a prime manifold whose first betti number is 1, and such manifold has left-orderable fundamental group [4, Corollary 3.4]. Furthermore, the same conclusion holds for 4-surgery on twist knots [16]. Hence we will handle the case where $0 < r < 4$ in this paper.

2. KNOT GROUP AND REPRESENTATIONS

Let K be the knot 5_2 in the knot table ([14]). See Figure 1. This knot is the two-bridge knot corresponding to the rational number $3/7$. In this diagram, K bounds a once-punctured Klein bottle, as seen from the checkerboard coloring, whose boundary slope is 4. In fact, 4-surgery on K gives a toroidal manifold, and 1, 2 and 3-surgeries give small Seifert fibered manifolds ([5]).

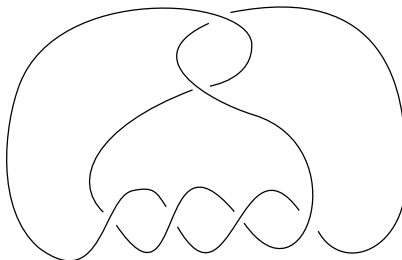


FIGURE 1.

Let M be the knot exterior of K . It is well known that the knot group $G = \pi_1(M)$ has a presentation $\langle x, y \mid wx = yw \rangle$, where x and y are meridians and $w = xyx^{-1}y^{-1}xy$. Also, a (preferred) longitude λ is given by $x^{-4}w^*w$, where $w^* = yxy^{-1}x^{-1}yx$ corresponds to the reverse word of w . (These facts are easily obtained from Schubert's normal form of the knot [15].)

Let $s > 0$ be a real number, and let $T = \frac{2+3s+2s^2+\sqrt{s^2+4}}{2s}$. Then it is easy to see that $T > 4$. Also, let $t = \frac{T+\sqrt{T^2-4}}{2}$. Then, $t > 3$ and

$$(2.1) \quad t = \frac{2 + 3s + 2s^2 + \sqrt{s^2 + 4} + \sqrt{(2 + 3s + 2s^2 + \sqrt{s^2 + 4})^2 - 16s^2}}{4s}.$$

Let $\phi = s(t+t^{-1})^2 - (2s^2+3s+2)(t+t^{-1}) + s^3+3s^2+4s+3$. Since $t+t^{-1} = T$, $\phi = sT^2 - (2s^2+3s+2)T + s^3+3s^2+4s+3$. If we solve the equation $\phi = 0$ with respect to T , we obtain the expression of T in terms of s as above. Thus $\phi = 0$ holds.

We now examine some limits, which will be necessary later.

- Lemma 2.1.** (1) $\lim_{s \rightarrow +0} t = \infty$,
 (2) $\lim_{s \rightarrow +0} st = 2$,
 (3) $t - s > 2$ and $\lim_{s \rightarrow \infty} (t - s) = 2$,
 (4) $\lim_{s \rightarrow \infty} s/t = 1$,
 (5) $\lim_{s \rightarrow \infty} s(t - s - 2) = 0$,
 (6) $\lim_{s \rightarrow \infty} t(t - s - 2) = 0$.

Proof. (1) and (2) are obvious from (2.1). For (3),

$$t - s = \frac{2 + 3s + \sqrt{s^2 + 4} + \left(\sqrt{(2 + 3s + 2s^2 + \sqrt{s^2 + 4})^2 - 16s^2} - 2s^2 \right)}{4s}$$

shows $t - s > 0$, since $(2 + 3s + 2s^2 + \sqrt{s^2 + 4})^2 - 16s^2 > 4s^4$. The second conclusion follows from

$$\lim_{s \rightarrow \infty} \frac{2 + 3s + \sqrt{s^2 + 4}}{4s} = 1, \quad \lim_{s \rightarrow \infty} \frac{\sqrt{(2 + 3s + 2s^2 + \sqrt{s^2 + 4})^2 - 16s^2} - 2s^2}{4s} = 1.$$

A direct calculation shows (4).

For (5),

$$4s(t - s - 2) - 2 = \left(\sqrt{(2 + 3s + 2s^2 + \sqrt{s^2 + 4})^2 - 16s^2} + \sqrt{s^2 + 4} \right) - (2s^2 + 5s).$$

Since the right hand side converges to -2 , we have $\lim_{s \rightarrow \infty} s(t - s - 2) = 0$.

From (3), an inequality $s + 2 < t < s + 3$ holds for sufficiently large s . Then $(s+2)(t-s-2) < t(t-s-2) < (s+3)(t-s-2)$. Hence (3) and (5) imply (6). \square

Let $\rho_s : G \rightarrow SL_2(\mathbb{R})$ be the representation defined by the correspondence

$$(2.2) \quad \rho_s(x) = \begin{pmatrix} \sqrt{t} & 0 \\ 0 & \frac{1}{\sqrt{t}} \end{pmatrix}, \quad \rho_s(y) = \begin{pmatrix} \frac{t-s-1}{\sqrt{t}-\frac{1}{\sqrt{t}}} & \frac{s}{(\sqrt{t}-\frac{1}{\sqrt{t}})^2} - 1 \\ -s & \frac{s+1-\frac{1}{t}}{\sqrt{t}-\frac{1}{\sqrt{t}}} \end{pmatrix}.$$

Here, we remain using the variable t to reduce the complexity. By using the fact that s and t satisfies the equation $\phi = 0$, we can check $\rho_s(wx) = \rho_s(yw)$ by a direct calculation. Hence the correspondence on x and y above gives a homomorphism from G to $SL_2(\mathbb{R})$. In addition, $\rho_s(xy) \neq \rho_s(yx)$, and so ρ_s has the non-abelian image.

Remark 2.2. This representation of G comes from that in [10, p.786]. The polynomial ϕ corresponds to the Riley polynomial [13].

Lemma 2.3. *For a longitude λ , $\rho_s(\lambda)$ is diagonal, and its $(1, 1)$ -entry is a positive real number.*

Proof. Note that $\rho_s(x)$ is diagonal and $\rho_s(x) \neq \pm I$. The fact that $\rho_s(x)$ commutes with $\rho_s(\lambda)$ easily implies that $\rho_s(\lambda)$ is also diagonal. (This can also be seen from a direct calculation of $\rho_s(\lambda)$, by using $\phi(s, t) = 0$.)

A direct calculation gives the $(1, 1)$ -entry

$$(2.3) \quad \frac{1}{(t-1)^2 t^5} \left(s(1 - (2+s)t + t^2) (s - (2+2s+s^2)t + (1+s)t^2)^2 \right. \\ \left. + (1+s-t)^2 t^3 (s - (1+s)^2 t + st^2)^2 \right)$$

of $\rho_s(\lambda)$. Thus it is enough to show that $1 - (2+s)t + t^2 > 0$. This is equivalent to the inequality $T > 2+s$, which is clear from $T = \frac{2+3s+2s^2+\sqrt{s^2+4}}{2s}$. \square

Let $r = p/q$ be a rational number, and let $M(r)$ denote the resulting manifold by r -filling on the knot exterior M of K . In other words, $M(r)$ is obtained by attaching a solid torus V to M along their boundaries so that the loop $x^p \lambda^q$ bounds a meridian disk of V .

Clearly, $\rho_s : G \rightarrow SL_2(\mathbb{R})$ induces a homomorphism $\pi_1(M(r)) \rightarrow SL_2(\mathbb{R})$ if and only if $\rho_s(x)^p \rho_s(\lambda)^q = I$. Since both of $\rho_s(x)$ and $\rho_s(\lambda)$ are diagonal, this is equivalent to the equation

$$(2.4) \quad A_s^p B_s^q = 1,$$

where A_s and B_s are the $(1, 1)$ -entries of $\rho_s(x)$ and $\rho_s(\lambda)$, respectively. We remark that $A_s = \sqrt{t}$ is a positive real number, so is B_s by Lemma 2.3. The equation (2.4) is furthermore equivalent to

$$(2.5) \quad -\frac{\log B_s}{\log A_s} = \frac{p}{q}.$$

Let $g : (0, \infty) \rightarrow \mathbb{R}$ be a function defined by

$$g(s) = -\frac{\log B_s}{\log A_s}.$$

Lemma 2.4. *The image of g contains an open interval $(0, 4)$.*

Proof. First, we show

$$\lim_{s \rightarrow +0} g(s) = 0.$$

Since $\lim_{s \rightarrow +0} \log A_s = \infty$, it is enough to show that $\lim_{s \rightarrow +0} B_s = 1$. We decompose B_s , given in (2.3), as

$$(2.6) \quad B_s = \frac{s}{t-1} \frac{1 - (2+s)t + t^2}{(t-1)t} \left(\frac{s - (2+2s+s^2)t + (1+s)t^2}{t^2} \right)^2 \\ + \left(\frac{1+s-t}{t-1} \right)^2 \left(\frac{s - (1+s)^2 t + st^2}{t} \right)^2.$$

From Lemma 2.1, $\lim_{s \rightarrow +0} t = \infty$ and $\lim_{s \rightarrow +0} st = 2$. These give

$$\lim_{s \rightarrow +0} \frac{s}{t-1} = 0, \quad \lim_{s \rightarrow +0} \frac{1 - (2+s)t + t^2}{(t-1)t} = 1,$$

$$\lim_{s \rightarrow +0} \frac{s - (2+2s+s^2)t + (1+s)t^2}{t^2} = 1, \quad \lim_{s \rightarrow +0} \frac{1+s-t}{t-1} = -1,$$

and

$$\lim_{s \rightarrow +0} \frac{s - (1+s)^2t + st^2}{t} = 1.$$

Thus we have $\lim_{s \rightarrow +0} B_s = 0$.

Second, we show

$$\lim_{s \rightarrow \infty} g(s) = 4.$$

Let N be the numerator of B_s shown in (2.3). Then

$$\frac{\log B_s}{\log A_s} = \frac{2 \log N}{\log t} - \frac{2 \log(t-1)^2 t^5}{\log t}.$$

Claim 2.5. $\lim_{s \rightarrow \infty} Nt^{-5} = 1$.

Proof of Claim 2.5. From Lemma 2.1, $\lim_{s \rightarrow \infty} s/t = 1$, and $\lim_{s \rightarrow \infty} (1+s-t) = -1$.

We have

$$\begin{aligned} 1 - (2+s)t + t^2 &= t(t-s-2) + 1, \\ \frac{s - (1+s)^2t + st^2}{t} &= \frac{s}{t} + s(t-s-2) - 1, \\ \frac{s - (2+2s+s^2)t + (1+s)t^2}{t^2} &= \frac{1}{t} \cdot \frac{s - (1+s)^2t + st^2}{t} - \frac{1}{t} + 1. \end{aligned}$$

Hence Lemma 2.1 implies

$$\lim_{s \rightarrow \infty} (1 - (2+s)t + t^2) = \lim_{s \rightarrow \infty} \frac{s - (2+2s+s^2)t + (1+s)t^2}{t^2} = 1,$$

$$\lim_{s \rightarrow \infty} \frac{s - (1+s)^2t + st^2}{t} = 0.$$

Combining these, we have $\lim_{s \rightarrow \infty} Nt^{-5} = 1$. □

Thus we have $\lim_{s \rightarrow \infty} (\log N - 5 \log t) = 0$. Then

$$\lim_{s \rightarrow \infty} \frac{\log N}{\log t} = 5.$$

Clearly,

$$\lim_{t \rightarrow \infty} \frac{\log(t-1)^2 t^5}{\log t} = 7.$$

Hence we have $\lim_{s \rightarrow \infty} g(s) = 4$. □

3. THE UNIVERSAL COVERING GROUP OF $SL_2(\mathbb{R})$

Let

$$SU(1, 1) = \left\{ \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} \mid |\alpha|^2 - |\beta|^2 = 1 \right\}$$

be the special unitary group over \mathbb{C} of signature $(1, 1)$. It is well known that $SU(1, 1)$ is conjugate to $SL_2(\mathbb{R})$ in $GL_2(\mathbb{C})$. The correspondence is given by $\psi : SL_2(\mathbb{R}) \rightarrow SU(1, 1)$, sending $A \mapsto JAJ^{-1}$, where

$$J = \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}.$$

Thus

$$\psi : \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} \frac{a+d+(b-c)i}{2} & \frac{a-d-(b+c)i}{2} \\ \frac{a-d+(b+c)i}{2} & \frac{a+d-(b-c)i}{2} \end{pmatrix}.$$

There is a parametrization of $SU(1, 1)$ by (γ, ω) where $\gamma = \beta/\alpha$ and $\omega = \arg \alpha$ defined mod 2π (see [1]). Thus $SU(1, 1) = \{(\gamma, \omega) \mid |\gamma| < 1, -\pi \leq \omega < \pi\}$. Topologically, $SU(1, 1)$ is an open solid torus $\Delta \times S^1$, where $\Delta = \{\gamma \in \mathbb{C} \mid |\gamma| < 1\}$. The group operation is given by $(\gamma, \omega)(\gamma', \omega') = (\gamma'', \omega'')$, where

$$(3.1) \quad \gamma'' = \frac{\gamma' + \gamma e^{-2i\omega'}}{1 + \gamma \bar{\gamma}' e^{-2i\omega'}},$$

$$(3.2) \quad \omega'' = \omega + \omega' + \frac{1}{2i} \log \frac{1 + \gamma \bar{\gamma}' e^{-2i\omega'}}{1 + \bar{\gamma} \gamma' e^{2i\omega'}}.$$

These equations come from the matrix operation. Here, the logarithm function is defined by its principal value and ω'' is defined by mod 2π . The identity element is $(0, 0)$, and the correspondence between $\begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix}$ and (γ, ω) gives an isomorphism.

Now, the universal covering group $\widetilde{SL_2(\mathbb{R})}$ of $SU(1, 1)$ can be described as

$$\widetilde{SL_2(\mathbb{R})} = \{(\gamma, \omega) \mid |\gamma| < 1, -\infty < \omega < \infty\}.$$

Thus $\widetilde{SL_2(\mathbb{R})}$ is homeomorphic to $\Delta \times \mathbb{R}$. The group operation is given by (3.1) and (3.2) again, but ω'' is not mod 2π anymore.

Let $\Phi : \widetilde{SL_2(\mathbb{R})} \rightarrow SL_2(\mathbb{R})$ be the covering projection. Then it is obvious that $\ker \Phi = \{(0, 2m\pi) \mid m \in \mathbb{Z}\}$.

Lemma 3.1. *The subset $(-1, 1) \times \{0\}$ of $\widetilde{SL_2(\mathbb{R})}$ forms a subgroup.*

Proof. From (3.1) and (3.2), it is straightforward to see that $(-1, 1) \times \{0\}$ is closed under the group operation. For $(\gamma, 0) \in (-1, 1) \times \{0\}$, its inverse is $(-\gamma, 0)$. \square

For the representation $\rho_s : G \rightarrow SL_2(\mathbb{R})$ defined by (2.2),

$$(3.3) \quad \psi(\rho_s(x)) = \frac{1}{2\sqrt{t}} \begin{pmatrix} t+1 & t-1 \\ t-1 & t+1 \end{pmatrix} \in SU(1, 1).$$

Thus $\psi(\rho_s(x))$ corresponds to $(\gamma_x, 0)$, where $\gamma_x = \frac{t-1}{t+1}$.

Also, for a longitude λ ,

$$\psi(\rho_s(\lambda)) = \frac{1}{2} \begin{pmatrix} B_s + \frac{1}{B_s} & B_s - \frac{1}{B_s} \\ B_s - \frac{1}{B_s} & B_s + \frac{1}{B_s} \end{pmatrix}, B_s > 0$$

from Lemma 2.3. Thus $\psi(\rho_s(\lambda))$ corresponds to $(\gamma_\lambda, 0)$, where $\gamma_\lambda = \frac{B_s^2 - 1}{B_s^2 + 1}$.

4. PROOF OF THEOREM

As the knot exterior M satisfies $H^2(M; \mathbb{Z}) = 0$, any $\rho_s : G \rightarrow SL_2(\mathbb{R})$ lifts to a representation $\tilde{\rho} : G \rightarrow \widetilde{SL_2(\mathbb{R})}$ [8]. Moreover, any two lifts $\tilde{\rho}$ and $\tilde{\rho}'$ are related as follows:

$$\tilde{\rho}'(g) = h(g)\tilde{\rho}(g),$$

where $h : G \rightarrow \ker \Phi \subset \widetilde{SL_2(\mathbb{R})}$. Since $\ker \Phi = \{(0, 2m\pi) \mid m \in \mathbb{Z}\}$ is isomorphic to \mathbb{Z} , the homomorphism h factors through $H_1(M)$, so it is determined only by the value $h(x)$ of a meridian x (see [10]).

The following result is the key in [3], which is originally claimed in [10], for the figure eight knot. Our proof most follows that of [3], but it is much simpler, because of the values of $\psi(\rho_s(x))$ and $\psi(\rho_s(\lambda))$, which are calculated in Section 3.

Lemma 4.1. *Let $\tilde{\rho} : G \rightarrow \widetilde{SL_2(\mathbb{R})}$ be a lift of ρ_s . Then replacing $\tilde{\rho}$ by a representation $\tilde{\rho}' = h \cdot \tilde{\rho}$ for some $h : G \rightarrow \widetilde{SL_2(\mathbb{R})}$, we can suppose that $\tilde{\rho}(\pi_1(\partial M))$ is contained in the subgroup $(-1, 1) \times \{0\}$ of $\widetilde{SL_2(\mathbb{R})}$.*

Proof. Since $\Phi(\tilde{\rho}(\lambda)) = (\gamma_\lambda, 0)$, $\gamma_\lambda \in (-1, 1)$, $\tilde{\rho}(\lambda) = (\gamma_\lambda, 2j\pi)$ for some j . On the other hand, λ is a commutator, because our knot is genus one. Therefore the inequality (5.5) of [17] implies $-3\pi/2 < 2j\pi < 3\pi/2$. Thus we have $\tilde{\rho}(\lambda) = (\gamma_\lambda, 0)$.

Similarly, $\tilde{\rho}(x) = (\gamma_x, 2\ell\pi)$ for some ℓ , where $\gamma_x \in (-1, 1)$. Let us choose $h : G \rightarrow \widetilde{SL_2(\mathbb{R})}$ so that $h(x) = (0, -2\ell\pi)$. Set $\tilde{\rho}' = h \cdot \tilde{\rho}$. Then a direct calculation shows that $\tilde{\rho}'(x) = (\gamma_x, 0)$ and $\tilde{\rho}'(\lambda) = (\gamma_\lambda, 0)$. Since x and λ generate the peripheral subgroup $\pi_1(\partial M)$, the conclusion follows from these. \square

Proof of Theorem 1.1. Let $r = p/q \in (0, 4)$. By Lemma 2.4, we can fix s so that $g(s) = r$. Choose a lift $\tilde{\rho}$ of ρ_s so that $\tilde{\rho}(\pi_1(\partial M)) \subset (-1, 1) \times \{0\}$. Then $\rho_s(x^p \lambda^q) = I$, so $\Phi(\tilde{\rho}(x^p \lambda^q)) = I$. This means that $\tilde{\rho}(x^p \lambda^q)$ lies in $\ker \Phi = \{(0, 2m\pi) \mid m \in \mathbb{Z}\}$. Hence $\tilde{\rho}(x^p \lambda^q) = (0, 0)$. Then $\tilde{\rho}$ can induce a homomorphism $\pi_1(M(r)) \rightarrow \widetilde{SL_2(\mathbb{R})}$ with non-abelian image. Recall that $\widetilde{SL_2(\mathbb{R})}$ is left-orderable [2]. Since $M(r)$ is irreducible [9], $\pi_1(M(r))$ is left-orderable by [4, Theorem 1.1]. This completes the proof. \square

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